

Attack trees: meaning, analysis, and correctness

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GdR Security, WG FM, January 2020



It's been already 20 years!

Schneier on Security



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Attack Trees

B. Schneier

Dr. Dobb's Journal, December 1999.

Modeling security threats

By **Bruce Schneier**

Few people truly understand computer security, as illustrated by computer-security company marketing literature that touts "hacker proof software," "triple-DES security," and the like. In truth, unbreakable security is broken all the time, often in ways its designers never imagined. Seemingly strong cryptography gets broken, too. Attacks thought to be beyond the ability of mortal men become commonplace. And as newspapers report security bug after security bug, it becomes increasingly clear that the term "security" doesn't have meaning unless also you know things like "Secure from whom?" or "Secure for how long?"

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About Bruce Schneier



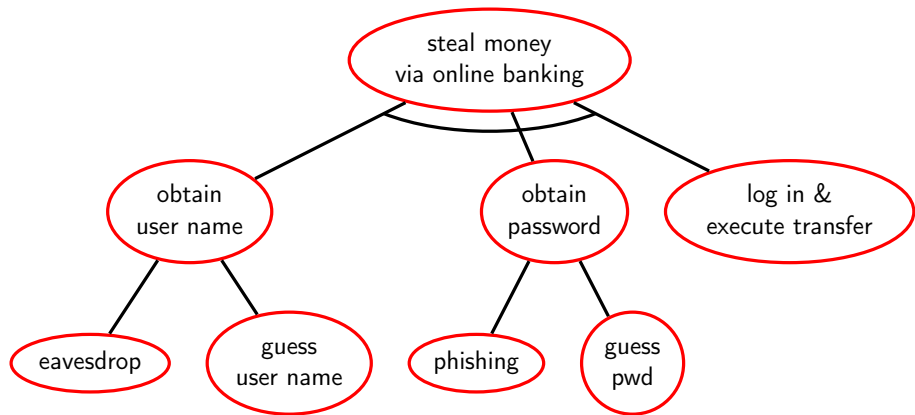
Outline

- 1 Attack trees
- 2 Repeated labels
- 3 State-based attack trees
- 4 There is much more going on

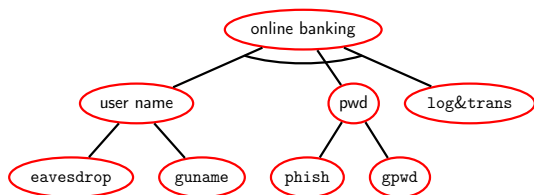
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An attack tree



Attacks



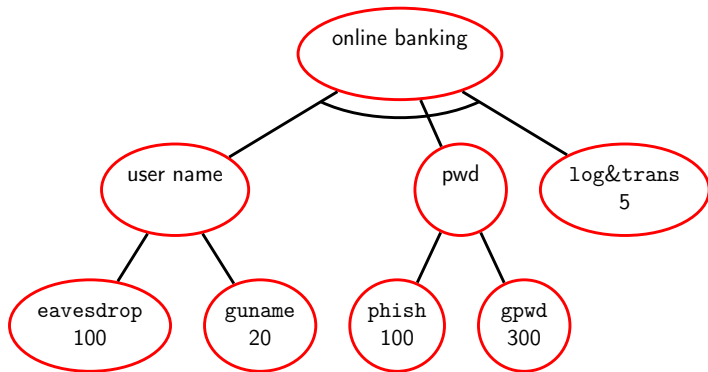
{eavesdrop, phish, log&trans}

{guname, phish, log&trans}

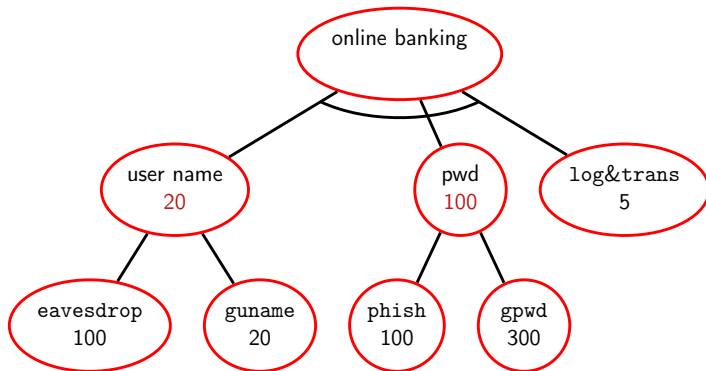
{eavesdrop, gpwd, log&trans}

{guname, gpwd, log&trans}

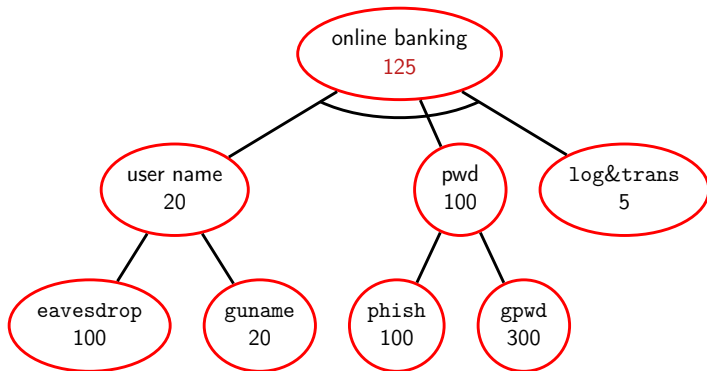
Bottom-up for min time



Bottom-up for min time



Bottom-up for min time



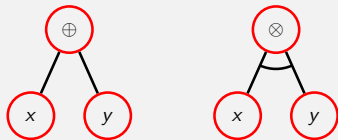
Bottom-up procedure formally

Basic assignment

Values assigned to the leaf nodes

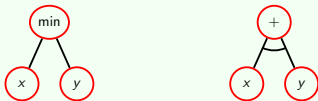
Attribute domain

Algebraic structure defining the propagation rules $A = (D, \oplus, \otimes)$



Example (Minimal time of attacking)

$$A_{\text{min_time}} = (\mathbb{N}, \min, +)$$



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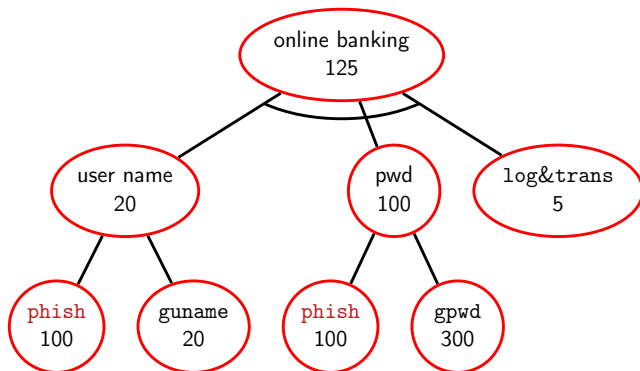
Reference

Wojciech Wideł and Barbara Kordy (Fila)

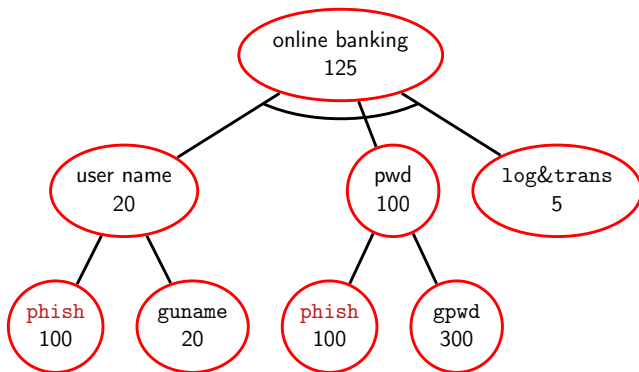
On quantitative analysis of attack-defense trees with repeated labels

POST 2018

A well-known problem



A well-known problem

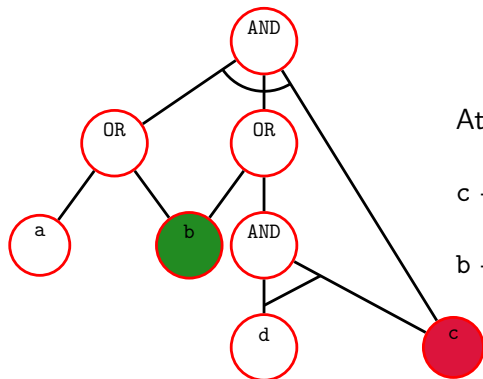


Overestimated!

Nodes with the same labels

Clones

Nodes representing the same instance of an action



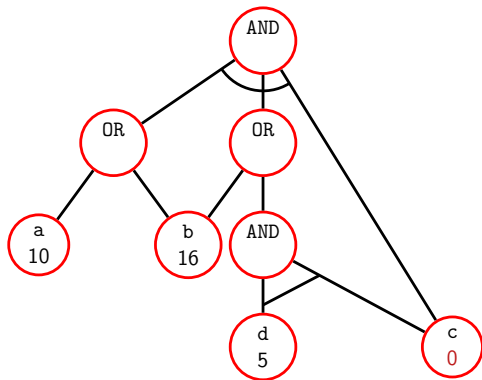
Attacks: $\{b, c\}, \{a, c, d\}$

c - **necessary clone**

b - **optional clone**

Neutralize necessary clones

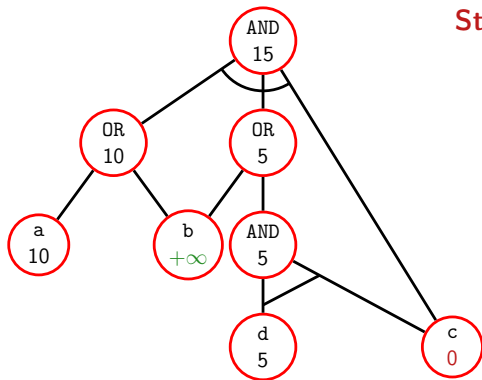
Step₀: $\text{time}'(c) := 0$



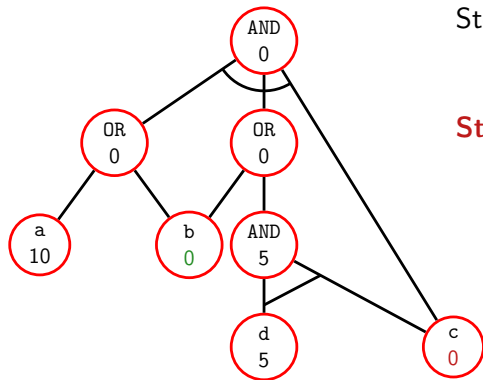
Play with the values of optional clones

Step₀: $\text{time}'(c) := 0$

Step₁: $\text{time}'(b) := +\infty$
 $\text{res}_1 = 15$



Play with the values of optional clones

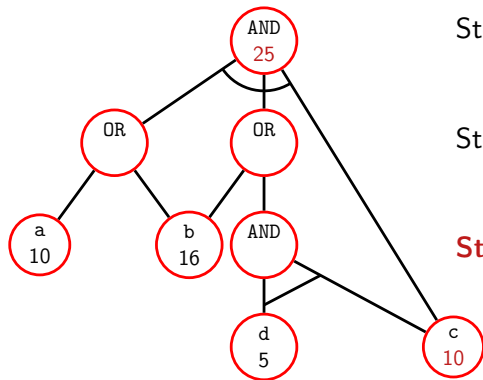


Step₀: $\text{time}'(c) := 0$

Step₁: $\text{time}'(b) := +\infty$
 $\text{res}_1 = 15$

Step₂: $\text{time}'(b) := 0$
 $\text{res}_2 = 0 + \text{time}(b) = 16$

Combine the results



Step₀: $\text{time}'(c) := 0$

Step₁: $\text{time}'(b) := +\infty$
 $\text{res}_1 = 15$

Step₂: $\text{time}'(b) := 0$
 $\text{res}_2 = 16$

Step₃: $\text{res} = \min\{15, 16\} + 10 = 25$

Algorithm for minimal time

Input: Attack tree t , $(\mathbb{R}^+, \min, +)$, basic assignment of time

Output: Minimal time of attacking

- 1: $\mathcal{C}_N \leftarrow$ necessary clones
- 2: $\mathcal{C}_O \leftarrow$ optional clones
- 3: $\text{time}'(b) \leftarrow 0$ for $b \in \mathcal{C}_N$
- 4: **for** every subset $\mathcal{C} \subseteq \mathcal{C}_O$ **do**
- 5: $\text{time}'(b) \leftarrow +\infty$ for every $b \in \mathcal{C}$
- 6: $\text{time}'(b) \leftarrow 0$ for every $b \in \mathcal{C}_O \setminus \mathcal{C}$
- 7: $r_{\mathcal{C}} \leftarrow \text{time}_B(t, \text{time}') + \sum_{b \in \mathcal{C}_O \setminus \mathcal{C}} \text{time}(b)$
- 8: **end for**
- 9: **return** $\min_{\mathcal{C} \subseteq \mathcal{C}_O} r_{\mathcal{C}} + (\sum_{b \in \mathcal{C}_N} \text{time}(b))$

Algorithm for minimal time

Input: Attack tree t , $(\mathbb{R}^+, \min, +)$, basic assignment of time

Output: Minimal time of attacking

1: $\mathcal{C}_N \leftarrow$ necessary clones

2: $\mathcal{C}_O \leftarrow$ optional clones

3: $\text{time}'(b) \leftarrow 0$ for $b \in \mathcal{C}_N$

// $0 = e_+$

4: **for** every subset $\mathcal{C} \subseteq \mathcal{C}_O$ **do**

5: $\text{time}'(b) \leftarrow +\infty$ for every $b \in \mathcal{C}$

// $+\infty = a_+ = e_{\min}$

6: $\text{time}'(b) \leftarrow 0$ for every $b \in \mathcal{C}_O \setminus \mathcal{C}$

7: $r_{\mathcal{C}} \leftarrow \text{time}_B(t, \text{time}') + \sum_{b \in \mathcal{C}_O \setminus \mathcal{C}} \text{time}(b)$

8: **end for**

9: **return** $\min_{\mathcal{C} \subseteq \mathcal{C}_O} r_{\mathcal{C}} + (\sum_{b \in \mathcal{C}_N} \text{time}(b))$

Algorithm in the general case

Input: Attack tree t , non-increasing attribute domain (D, \oplus, \otimes) ,
basic assignment for attribute α

Output: $A(t, \alpha)$

- 1: $\mathcal{C}_N \leftarrow$ necessary clones
- 2: $\mathcal{C}_O \leftarrow$ optional clones
- 3: $\alpha'(b) \leftarrow e_{\otimes}$ for $b \in \mathcal{C}_N$
- 4: **for** every subset $\mathcal{C} \subseteq \mathcal{C}_O$ **do**
- 5: $\alpha'(b) \leftarrow a_{\otimes}$ for every $b \in \mathcal{C}$
- 6: $\alpha'(b) \leftarrow e_{\otimes}$ for every $b \in \mathcal{C}_O \setminus \mathcal{C}$
- 7: $r_{\mathcal{C}} \leftarrow \alpha_B(t, \alpha') \otimes \bigotimes_{b \in \mathcal{C}_O \setminus \mathcal{C}} \alpha(b)$
- 8: **end for**
- 9: **return** $\bigoplus_{\mathcal{C} \subseteq \mathcal{C}_O} r_{\mathcal{C}} \otimes \left(\bigotimes_{b \in \mathcal{C}_N} \alpha(b) \right)$

Non-increasing attribute domain

Commutative idempotent semiring

An algebraic structure (D, \oplus, \otimes) where

- \oplus is **idempotent**
- \oplus and \otimes are **associative** and **commutative**
- \otimes **distributes** over \oplus
- absorbing element wrt \otimes is equal to the neutral element wrt \oplus
 $a_{\otimes} = e_{\oplus}$

Canonical partial order \preceq in an idempotent semiring: $x \preceq y$ iff $x \oplus y = y$

Non-increasing attribute domain

An attribute domain (D, \oplus, \otimes) where

- (D, \oplus, \otimes) is a commutative idempotent semiring
- $x \otimes y \preceq y$ (doing less is better)

Interesting attribute domains

Example (minimal time)

$(\mathbb{N}, \min, +)$

Example (maximal probability)

$([0, 1], \max, \cdot)$

Example (satisfiability)

$(\{T, F\}, \vee, \wedge)$

Example (required skills level)

(\mathbb{N}, \min, \max)

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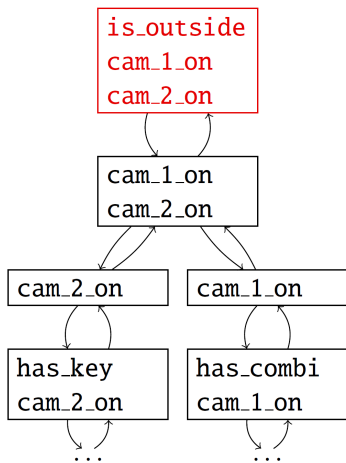
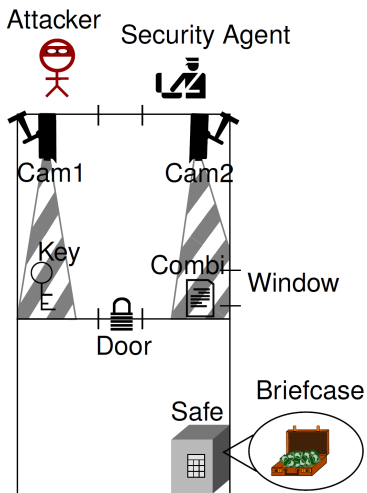
Reference

Maxime Audinot, Sophie Pinchinat, and Barbara Kordy (Fila)

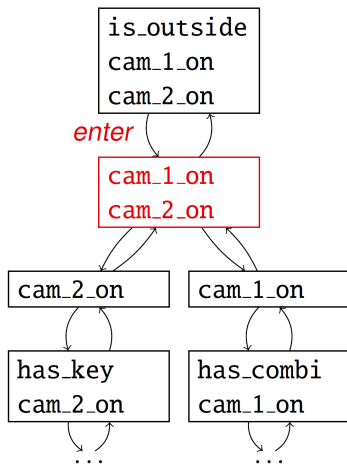
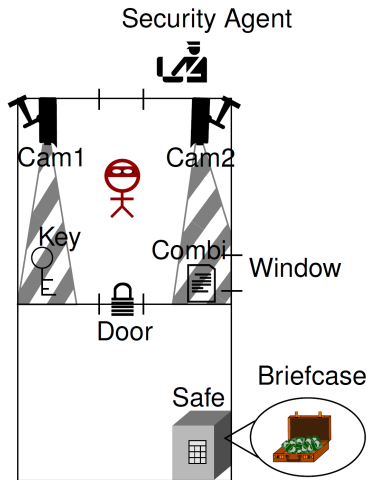
Is My Attack Tree Correct?

ESORICS 2017

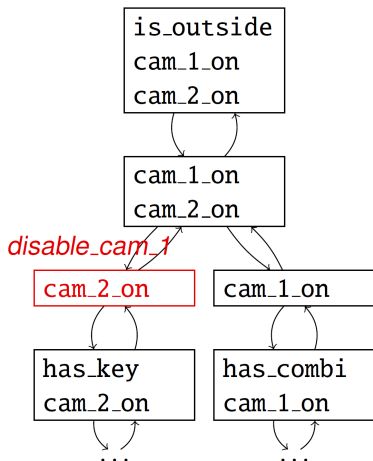
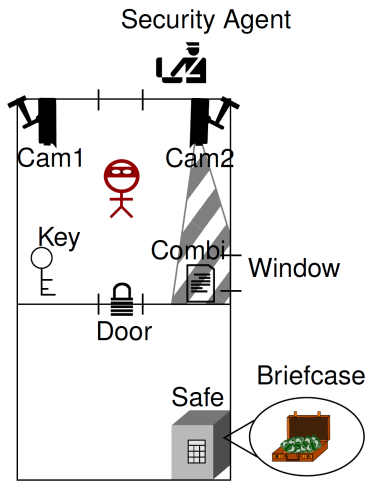
Modeling the system



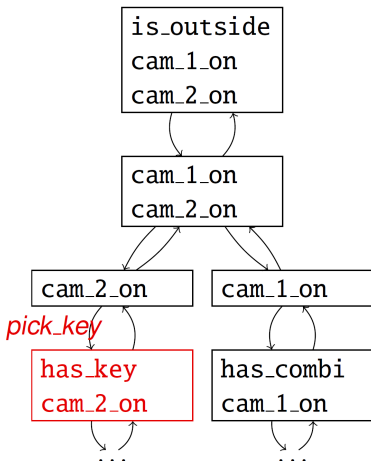
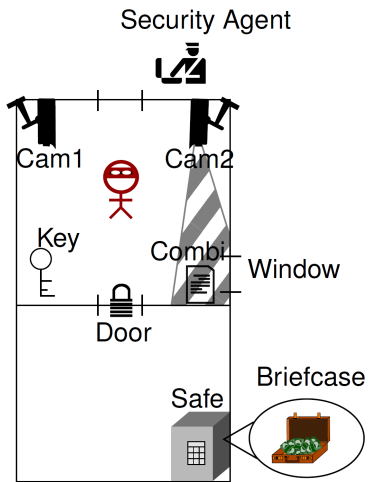
Modeling the system



Modeling the system



Modeling the system



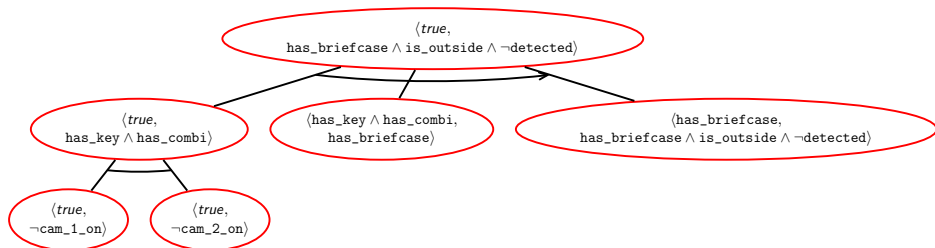
State-based attack tree

Goal $\langle \iota, \gamma \rangle$

- ι – precondition
- γ – postcondition

State-based attack tree grammar

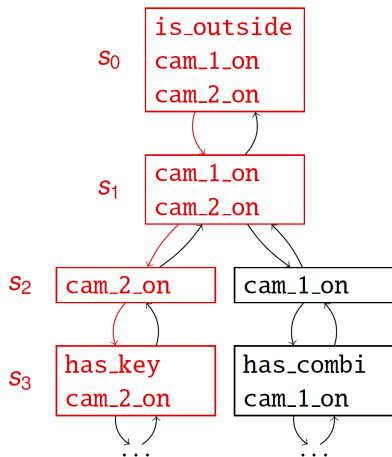
$\tau ::= \langle \iota, \gamma \rangle \mid \text{OP}(\tau_1, \dots, \tau_n)$
 where $\text{OP} \in \{\text{OR}, \text{AND}, \text{SAND}\}$



Paths in transition systems

Paths in the system Sys

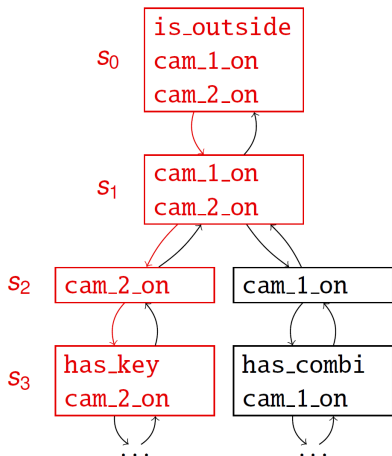
A path is a sequence of states $\pi = s_0 \dots s_n$ with $s_i \rightarrow s_{i+1}$ for all i .



Paths in transition systems

Paths in the system Sys

A path is a sequence of states $\pi = s_0 \dots s_n$ with $s_i \rightarrow s_{i+1}$ for all i .



Goal $\langle \iota, \gamma \rangle$ over Prop

- ι – precondition
- γ – postcondition

$$\pi = s_0 s_1 s_2 s_3$$

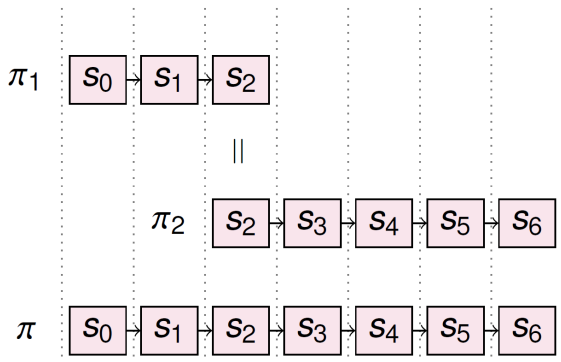
$$s_0 \models \text{is_outside}$$

$$s_3 \models \text{has_key}$$

π achieves $\langle \text{is_outside}, \text{has_key} \rangle$

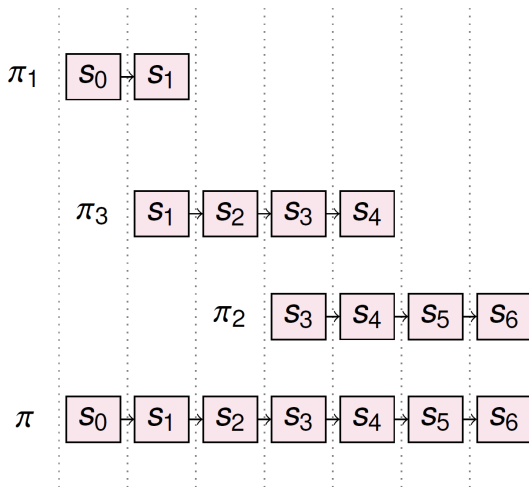
Sequential composition of paths •

$$\pi = \pi_1 \bullet \pi_2$$



Parallel composition of paths \mathbb{M}

$$\pi = \mathbb{M}(\pi_1, \pi_2, \pi_3)$$



Path semantics $[\tau]^{Sys}$

State-based attack trees formalized with sets of paths

Let Sys be a system.

The path semantics $[\cdot]^{Sys}$ is a set of paths in Sys , constructed as follows

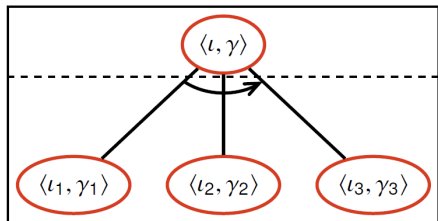
- $[\langle \iota, \gamma \rangle]^{Sys} = \{\pi \mid \pi \text{ achieves } \langle \iota, \gamma \rangle \text{ in } Sys\}$
- $[OR(\tau_1, \dots, \tau_n)]^{Sys} = [\tau_1]^{Sys} \cup \dots \cup [\tau_n]^{Sys}$
- $[SAND(\tau_1, \dots, \tau_n)]^{Sys} = [\tau_1]^{Sys} \bullet \dots \bullet [\tau_n]^{Sys}$
- $[AND(\tau_1, \dots, \tau_n)]^{Sys} = \mathbb{M}([\tau_1]^{Sys}, \dots, [\tau_n]^{Sys})$

Analysis of state-based attack trees

How can we exploit the path semantics to analyze state-based attack trees?

Refinement quality problem

Labels of intermediate nodes represent the history of refinement



$$\langle \iota, \gamma \rangle$$

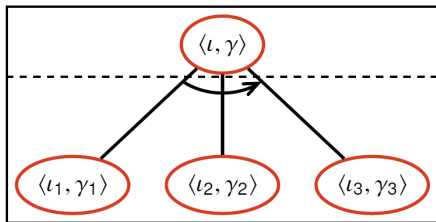
↓ refinement

$$OP(\langle \iota_1, \gamma_1 \rangle, \langle \iota_2, \gamma_2 \rangle, \langle \iota_3, \gamma_3 \rangle)$$

Objective

How well has a node been refined with respect to a given system?

Meet



$\exists \pi$ in Sys, such that

$$\pi \in [\langle \iota, \gamma \rangle]^{Sys}$$

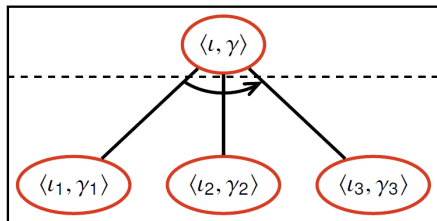
$$\pi \in [OP(\langle \iota_1, \gamma_1 \rangle, \langle \iota_2, \gamma_2 \rangle, \langle \iota_3, \gamma_3 \rangle)]^{Sys}$$

Meet property

There exists a common path achieving the node's goal and its refinement

$$[\langle \iota, \gamma \rangle]^{Sys} \cap [OP(\langle \iota_1, \gamma_1 \rangle, \langle \iota_2, \gamma_2 \rangle, \langle \iota_3, \gamma_3 \rangle)]^{Sys} \neq \emptyset$$

Match



$$[\langle \iota, \gamma \rangle]^{Sys}$$

|| Match

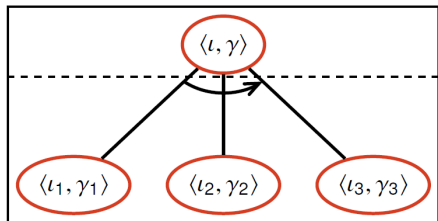
$$[OP(\langle \iota_1, \gamma_1 \rangle, \langle \iota_2, \gamma_2 \rangle, \langle \iota_3, \gamma_3 \rangle)]^{Sys}$$

Match property

Ideal situation – the node and its refinement model the same set of paths

$$[\langle \iota, \gamma \rangle]^{Sys} = [OP(\langle \iota_1, \gamma_1 \rangle, \langle \iota_2, \gamma_2 \rangle, \langle \iota_3, \gamma_3 \rangle)]^{Sys}$$

Under-Match



$$[\langle \iota, \gamma \rangle]^{Sys}$$

$\not\sqsubseteq$ Under-Match

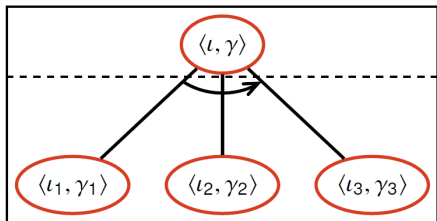
$$[OP(\langle \iota_1, \gamma_1 \rangle, \langle \iota_2, \gamma_2 \rangle, \langle \iota_3, \gamma_3 \rangle)]^{Sys}$$

Under-Match property

Forgotten attack scenarios – refinement models less paths

$$[\langle \iota, \gamma \rangle]^{Sys} \not\sqsupseteq [OP(\langle \iota_1, \gamma_1 \rangle, \langle \iota_2, \gamma_2 \rangle, \langle \iota_3, \gamma_3 \rangle)]^{Sys}$$

Over-Match



$$[\langle \iota, \gamma \rangle]^{Sys}$$

$\cap \dagger$ **Over-Match**

$$[OP(\langle \iota_1, \gamma_1 \rangle, \langle \iota_2, \gamma_2 \rangle, \langle \iota_3, \gamma_3 \rangle)]^{Sys}$$

Over-Match property

Extra attack scenarios – refinement models more paths

$$[\langle \iota, \gamma \rangle]^{Sys} \subsetneq [OP(\langle \iota_1, \gamma_1 \rangle, \langle \iota_2, \gamma_2 \rangle, \langle \iota_3, \gamma_3 \rangle)]^{Sys}$$

Complexity

	Meet	Under-Match	Over-Match	Match
OR	P	P	P	P
SAND	P	P	P	P
AND	NP-c	co-NP-c	co-NP	co-NP

High complexity is induced by the AND refinement (due to \wedge)

Witness of refinement property violation

Witness path generation by reduction to CTL model checking

Support for attack tree design

ATSyRA: Attack tree synthesis and risk analysis

ATSyRA studio tool <http://atsyra2.irisa.fr/>

- DSL for system specification
- Automated attack generation
- Attack tree refinement analysis



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Surveys

- Wojciech Wideł, Maxime Audinot, Barbara Fila, and Sophie Pinchinat. *Beyond 2014: formal methods for attack tree-based security modeling*. ACM Comput. Surv., 52(4):75:1–75:36, 2019.
- Barbara Kordy, Ludovic Piètre-Cambacédès, and Patrick Schweitzer. *Dag-based attack and defense modeling: Don't miss the forest for the attack trees*. Computer Science Review, 13–14:1–38, 2014.

Electronic versions available on

<http://people.irisa.fr/Barbara.Kordy/publications.php>

The most important open research problem

Automated generation
of large attack trees

GraMSec <http://www.gramsec.uni.lu/>

GraMSec 2014

The First International Workshop on
Graphical Models for Security
April 12, 2014, Grenoble, France
Co-located with [ETAPS 2014](#)

GraMSec 2015

The Second International Workshop on
Graphical Models for Security
Verona, Italy - July 13, 2015
Co-located with [CSE 2015](#)

GraMSec 2016

The Third International Workshop on
Graphical Models for Security
Lisbon, Portugal - June 27, 2016
Co-located with [CSE 2016](#)
GraMSec'16 post-proceedings are available online

GraMSec 2017

The Fourth International Workshop on
Graphical Models for Security
Santa Barbara, CA, USA - August 21, 2017
Co-located with [CSE 2017](#)
Camera ready version for post-proceedings due on September 27

GraMSec 2018

The Fifth International Workshop on
Graphical Models for Security
Oxford, UK - July 8, 2018
Co-located with [CSF 2018](#), in conjunction with [FLoC 2018](#)

GraMSec 2019

The Sixth International Workshop on
Graphical Models for Security
Hoboken, NJ, USA - June 24, 2019
Co-located with [CSF 2019](#)

The unique dissemination event for researchers and practitioners designing and using graphical approaches for modeling and analysis of security

Co-located with CSF

Do you have any questions?

Thank you for your attention